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Topcolor

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TOPCOLOR[1]

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We review a class of dynamical models in which top condensation occurs at the weak scale, giving rise to the large top quark mass and other phenomena. This typically requires a color embedding, $SU(3)_c \rightarrow SU(3)_1 \times SU(3)_2$, ergo "Topcolor." Topcolor suggests a novel route to technicolor models in which sequential quarks condense under the Topcolor interaction to break electroweak symmetries.

1 Top Quark Condensation

1.1 Preliminary

The top quark has arrived with a mass of ~ 175 GeV. This is remarkably large in comparison to all other known fundamental particles, and coincidently equals the natural scale of electroweak interactions, $1/\sqrt{2\sqrt{2}G_F}$. The top quark mass may be dynamically generated, analogous to a constituent quark mass in QCD. Then the relevant question becomes: "Are there new strong interactions that are revealing themselves through the large top mass?"

If the answer is no, then the most sensible scenario for physics beyond the standard model is something like the minimal supersymmetric standard model (MSSM). In this case the top quark mass is given by the infra–red renormalization group fixed point [1], and many of the successful predictions of unification are tied to this fact [2]. By itself, this is remarkable, because the infra–red renormalization group fixed point is really a consequence of the approximate scale invariance of the unified theory over the range of

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[1]Invited Talk presented at Strongly Coupled Gauge Theories '96, Nagoya, Nov. 1996

scales defining a desert, i.e., trace anomalies which violate scale invariance are proportional to beta–functions, and the fixed point corresponds to an approximate vanishing of the top–Yukawa beta function. This scale invariance must occur if there is a gauge hierarchy. *The top quark appears to be the only known particle which yields a nontrivial vanishing of its Higgs–Yukawa beta–function, ergo a nontrivial realization of the hierarchical scale invariance, in the context of the MSSM.*

On the other hand, if the answer is yes, then there are potentially new effects that are necessarily (1) strongly coupled and (2) generational, which will show up in studies of both top and bottom, and possibly other rare processes. Naturalness suggests that the scale of new dynamics must not be too far beyond the top quark mass. Indeed, the third generation is now being studied extensively at the high energies of the Fermilab Tevatron, and LEP. This is the first expedition into the multi–hundred GEV scale, and the possibility of new physics emerging in unexpected ways is not precluded. By "new strong dynamics in the top system" we typically mean some kind of top quark condensation, i.e., Cooper pairing of $\bar{t}_L$ and t_R to form a vacuum condensate. The original top quark condensation models tried to identify all of the electroweak symmetry breaking ESB with the formation of a dynamical top quark mass [3]. Top condensation models must either allow the scale of new dynamics, $\Lambda >> m_t$ with drastic fine-tuning, in which case the top mass is again controlled by the renormalization group fixed point [1], or invoke new dynamical mechanisms to try to obtain a natural scheme. A partial contribution to ESB by top condensation is therefore a logical possibility. [4] [5]

1.2 Models with Strong $U(1)$ Tilters (Topcolor I).

We consider the possibility that the top quark mass is large because it is a combination of a *dynamical condensate component*, $(1 - \epsilon)m_t$, generated by a new strong dynamics, together with a small *fundamental component*, ϵm_t, i.e, $\epsilon << 1$, generated by an extended Technicolor[6] (ETC) or Higgs sector. The new strong dynamics is assumed to be chiral–critically strong but spontaneously broken, perhaps by TC itself, at the scale ~ 1 TeV, and it is coupled preferentially to the third generation. The new strong dynamics therefore occurs primarily in interactions that involve $\bar{t}\bar{t}tt$, $\bar{t}t\bar{b}b$, and $\bar{b}b\bar{b}b$, while the ETC interactions of the form $\bar{t}t\overline{Q}Q$, where Q is a Techniquark, are relatively feeble.

Our basic assumptions leave little freedom of choice in the new dynamics.

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We assume a new class of Technicolor models incorporating "Topcolor". In Topcolor I the dynamics at the ~ 1 TeV scale involves the following structure (or a generalization thereof):[5] [7] [8]

$$SU(3)_1 \times SU(3)_2 \times U(1)_{Y1} \times U(1)_{Y2} \times SU(2)_L \to SU(3)_{QCD} \times U(1)_{EM} \quad (1)$$

where $SU(3)_1 \times U(1)_{Y1}$ ($SU(3)_2 \times U(1)_{Y2}$) generally couples preferentially to the third (first and second) generations. The $U(1)_{Yi}$ are just strongly rescaled versions of electroweak $U(1)_Y$. We remark that employing a new $SU(2)_{L,R}$ strong interaction in the third generation is also thinkable, but may be problematic due to potentially large instanton effects that violate $B + L$. We will not explore this latter possibility further.

The fermions are then assigned $(SU(3)_1, SU(3)_2, Y_1, Y_2)$ quantum numbers in the following way:

$$
\begin{aligned}
(t,b)_L &\sim (3,1,1/3,0) & (t,b)_R &\sim (3,1,(4/3,-2/3),0) \quad (2) \\
(\nu_\tau, \tau)_L &\sim (1,1,-1,0) & \tau_R &\sim (1,1,-2,0)
\end{aligned}
$$

$$
\begin{aligned}
(u,d)_L, \ (c,s)_L &\sim (1,3,0,1/3) & (u,d)_R, \ (c,s)_R &\sim (1,3,0,(4/3,-2/3)) \\
(\nu,\ell)_L \ \ell = e,\mu &\sim (1,1,0,-1) & \ell_R &\sim (1,1,0,-2)
\end{aligned}
$$

Topcolor must be broken, which we will assume is accomplished through an (effective) scalar field:

$$\Phi \sim (3, \bar{3}, y, -y) \quad (3)$$

When Φ develops a VEV, it produces the simultaneous symmetry breaking

$$SU(3)_1 \times SU(3)_2 \to SU(3)_{QCD} \quad \text{and} \quad U(1)_{Y1} \times U(1)_{Y2} \to U(1)_Y \quad (4)$$

The choice of y will be specified below.

$SU(3)_1 \times U(1)_{Y1}$ is assumed to be strong enough to form chiral condensates which will naturally be tilted in the top quark direction by the $U(1)_{Y1}$ couplings. The theory is assumed to spontaneously break down to ordinary QCD $\times U(1)_Y$ at a scale of ~ 1 TeV, before it becomes confining. The isospin splitting that permits the formation of a $\langle \bar{t}t \rangle$ condensate but disallows the $\langle \bar{b}b \rangle$ condensate is due to the $U(1)_{Yi}$ couplings. Since they are both larger than the ordinary hypercharge gauge coupling, no significant fine–tuning is needed in principle to achieve this symmetry breaking pattern. The b–quark mass in this scheme is then an interesting issue, arising

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from a combination of ETC effects and instantons in $SU(3)_1$. The θ–term in $SU(3)_1$ may manifest itself as the CP–violating phase in the CKM matrix. Above all, the new spectroscopy of such a system should begin to materialize indirectly in the third generation (e.g., in $Z \to \overline{b}b$) or perhaps at the Tevatron in top and bottom quark production. The symmetry breaking pattern outlined above will generically give rise to three (pseudo)–Nambu–Goldstone bosons $\tilde{\pi}^a$, or "top-pions", near the top mass scale. If the Topcolor scale is of the order of 1 TeV, the top-pions will have a decay constant of $f_\pi \approx 50$ GeV, and a strong coupling given by a Goldberger–Treiman relation, $g_{tb\pi} \approx m_t/\sqrt{2}f_\pi \approx 2.5$, potentially observable in $\tilde{\pi}^+ \to t+\overline{b}$ if $m_{\tilde{\pi}} > m_t+m_b$.

We assume that ESB can be primarily driven by a Higgs sector or Technicolor, with gauge group G_{TC}.[9] Technicolor can also provide condensates which generate the breaking of Topcolor to QCD and $U(1)_Y$, although this can also be done by a Higgs field. The coupling constants (gauge fields) of $SU(3)_1 \times SU(3)_2$ are respectively h_1 and h_2 ($A_{1\mu}^A$ and $A_{2\mu}^A$) while for $U(1)_{Y1} \times U(1)_{Y2}$ they are respectively q_1 and q_2, ($B_{1\mu}, B_{2\mu}$). The $U(1)_{Yi}$ fermion couplings are then $q_i\frac{Yi}{2}$, where $Y1, Y2$ are the charges of the fermions under $U(1)_{Y1}, U(1)_{Y2}$ respectively. A $(3,\overline{3}) \times (y,-y)$ Techni–condensate (or Higgs field) breaks $SU(3)_1 \times SU(3)_2 \times U(1)_{Y1} \times U(1)_{Y2} \to SU(3)_{QCD} \times U(1)_Y$ at a scale $\Lambda \gtrsim 240$ GeV, or it fully breaks $SU(3)_1 \times SU(3)_2 \times U(1)_{Y1} \times U(1)_{Y2} \times SU(2)_L \to SU(3)_{QCD} \times U(1)_{EM}$ at the scale $\Lambda_{TC} = 240$ GeV. Either scenario typically leaves a *residual global symmetry*, $SU(3)' \times U(1)'$, implying a degenerate, massive color octet of "colorons," B_μ^A, and a singlet heavy Z'_μ. The gluon A_μ^A and coloron B_μ^A (the SM $U(1)_Y$ field B_μ and the $U(1)'$ field Z'_μ), are then defined by orthogonal rotations with mixing angle θ (θ'):

$$h_1 \sin\theta = g_3; \qquad h_2 \cos\theta = g_3; \qquad \cot\theta = h_1/h_2; \qquad \frac{1}{g_3^2} = \frac{1}{h_1^2} + \frac{1}{h_2^2};$$

$$q_1 \sin\theta' = g_1; \qquad q_2 \cos\theta' = g_1; \qquad \cot\theta' = q_1/q_2;$$

$$\frac{1}{g_1^2} = \frac{1}{q_1^2} + \frac{1}{q_2^2}; \tag{5}$$

and g_3 (g_1) is the QCD ($U(1)_Y$) coupling constant at Λ_{TC}. adjusted to unity abelian $U(1)$ couplings is We ultimately demand $\cot\theta \gg 1$ and $\cot\theta' \gg 1$ to select the top quark direction for condensation. The masses of the degenerate octet of colorons and Z' are given by $M_B \approx g_3\Lambda/\sin\theta\cos\theta$ $M_{Z'} \approx g_1\Lambda/\sin\theta'\cos\theta'$. The usual QCD gluonic ($U(1)_Y$ electroweak) interactions are obtained for any quarks that carry either $SU(3)_1$ or $SU(3)_2$ triplet quantum numbers (or $U(1)_{Yi}$ charges).

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The coupling of the new heavy bosons Z' and B^A to fermions is then given by

$$\mathcal{L}_{Z'} = g_1 \cot \theta' (Z' \cdot J_{Z'}) \qquad \mathcal{L}_B = g_3 \cot \theta (B^A \cdot J_B^A) \tag{6}$$

where the currents $J_{Z'}$ and J_B in general involve all three generations of fermions

$$J_{Z'} = J_{Z',1} + J_{Z',2} + J_{Z',3} \qquad J_B = J_{B,1} + J_{B,2} + J_{B,3} \tag{7}$$

For the third generation the currents read explicitly (in a weak eigenbasis):

$$\begin{aligned}
J_{Z',3}^\mu &= \frac{1}{6}\bar{t}_L\gamma^\mu t_L + \frac{1}{6}\bar{b}_L\gamma^\mu b_L + \frac{2}{3}\bar{t}_R\gamma^\mu t_R - \frac{1}{3}\bar{b}_R\gamma^\mu b_R \\
&\quad -\frac{1}{2}\bar{\nu}_{\tau L}\gamma^\mu \nu_{\tau L} - \frac{1}{2}\bar{\tau}_L\gamma^\mu \tau_L - \bar{\tau}_R\gamma^\mu \tau_R
\end{aligned} \tag{8}$$

$$J_{B,3}^{A;\mu} = \bar{t}\gamma^\mu \frac{\lambda^A}{2} t + \bar{b}\gamma^\mu \frac{\lambda^A}{2} b \tag{9}$$

where λ^A is a Gell-Mann matrix acting on color indices. For the first two generations the expressions are similar, except for a suppression factor of $-\tan^2 \theta'$ ($-\tan^2 \theta$)

$$J_{Z',2}^\mu = -\tan^2 \theta' \left(\frac{1}{6}\bar{c}_L\gamma^\mu c_L + \frac{1}{6}\bar{s}_L\gamma^\mu s_L + \ldots \right) \tag{10}$$

$$J_{B,2}^\mu = -\tan^2 \theta \left(\bar{c}\gamma^\mu \frac{\lambda^A}{2} c + \bar{s}\gamma^\mu \frac{\lambda^A}{2} s \right) \tag{11}$$

with corresponding formulae applying to the first generation. Integrating out the heavy bosons Z' and B, these couplings give rise to effective low energy four fermion interactions. The effective Topcolor interaction of the third generation takes the form:

$$\mathcal{L}'_{TopC} = -\frac{2\pi\kappa}{M_B^2} \left(\bar{t}\gamma_\mu \frac{\lambda^A}{2} t + \bar{b}\gamma_\mu \frac{\lambda^A}{2} b \right) \left(\bar{t}\gamma^\mu \frac{\lambda^A}{2} t + \bar{b}\gamma^\mu \frac{\lambda^A}{2} b \right). \tag{12}$$

where

$$\kappa_1 = \frac{g_1^2 \cot^2 \theta'}{4\pi} \qquad \kappa = \frac{g_3^2 \cot^2 \theta}{4\pi} \tag{13}$$

This interaction is attractive in the color-singlet $\bar{t}t$ and $\bar{b}b$ channels and invariant under color $SU(3)$ and $SU(2)_L \times SU(2)_R \times U(1) \times U(1)$ where $SU(2)_R$ is the custodial symmetry of the electroweak interactions.

In addition to the Topcolor interaction, we have the $U(1)_{Y1}$ interaction (which breaks custodial $SU(2)_R$):

$$\mathcal{L}'_{Y1} = -\frac{2\pi\kappa_1}{M^2_{Z'}} \left(\frac{1}{6}\bar{\psi}_L \gamma_\mu \psi_L + \frac{2}{3}\bar{t}_R \gamma_\mu t_R - \frac{1}{3}\bar{b}_R \gamma_\mu b_R - \frac{1}{2}\bar{\ell}_L \gamma_\mu \ell_L - \bar{\tau}_R \gamma_\mu \tau_R \right)^2 \tag{14}$$

where $\psi_L = (t,b)_L$, $\ell_L = (\nu_\tau, \tau)_L$ and κ_1 is assumed to be $O(1)$. (A small value for κ_1 would signify fine-tuning and may be phenomenologically undesirable.)

The attractive Topcolor interaction, for sufficiently large κ, can trigger the formation of a low energy condensate, $\left\langle \bar{t}t + \bar{b}b \right\rangle$, which would break $SU(2)_L \times SU(2)_R \times U(1)_Y \to U(1) \times SU(2)_c$, where $SU(2)_c$ is a global custodial symmetry. On the other hand, the $U(1)_{Y1}$ force is attractive in the $\bar{t}t$ channel and repulsive in the $\bar{b}b$ channel. Thus, we can have in concert critical and subcritical values of the combinations:

$$\kappa + \frac{2\kappa_1}{9N_c} > \kappa_{crit}; \qquad \kappa_{crit} > \kappa - \frac{\kappa_1}{9N_c}; \tag{15}$$

Here N_c is the number of colors. It should be mentioned that this analysis is performed in the context of a large-N_c approximation. The leading isospin-breaking effects are kept even though they are $\mathcal{O}(1/N_c)$. The critical coupling, in this approximation, is given by $\kappa_{crit} = 2\pi/N_c$. In what follows, we will not make explicit the N_c dependence, but rather take $N_c = 3$. We would expect the cut–off for integrals in the usual Nambu–Jona-Lasinio (NJL) gap equation for $SU(3)_{TopC}$ ($U(1)_{Y1}$) to be $\sim M_B$ ($\sim M_{Z'}$). Hence, these relations define criticality conditions irrespective of $M_{Z'}/M_B$. This leads to "tilted" gap equations in which the top quark acquires a constituent mass, while the b quark remains massless. Given that both κ and κ_1 are large there is no particular fine–tuning occuring here, only "rough–tuning" of the desired tilted configuration. Of course, the NJL approximation is crude, but as long as the associated phase transitions of the real strongly coupled theory are approximately second order, analogous rough–tuning in the full theory is possible.

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Fig.(1) The full phase diagram of the Topcolor I model.

The full phase diagram of the model is shown in Fig. 1.[10] The criticality conditions (15) define the allowed region in the κ_1–κ plane in the form of the two straight solid lines intersecting at $(\kappa_1 = 0, \kappa = \kappa_{crit})$. To the left of these lines lies the symmetric phase, in between them the region where only a $\langle \bar{t}t \rangle$ condensate forms and to the right of them the phase where both $\langle \bar{t}t \rangle$ and $\langle \bar{b}b \rangle$ condensates arise. The horizontal line marks the region above which κ_1 makes the $U(1)_{Y1}$ interaction strong enough to produce a $\langle \bar{\tau}\tau \rangle$ condensate. (This line is meant only as an indication, as the fermion-bubble (large-N_c) approximation, which we use, evidently fails for leptons.) There is an additional constraint from the measurement of $\Gamma(Z \to \tau^+\tau^-)$, confining the allowed region to the one below the solid curve. This curve corresponds to a 2σ discrepancy between the Topcolor prediction and the measured value of this width. In the allowed region a top condensate alone forms. The constraints favor a strong $SU(3)_{TopC}$ coupling and a relatively weaker $U(1)_{Y1}$ coupling.

1.3 Anomaly–Free Models Without Strong $U(1)$ Tilters (Topcolor II).

The strong $U(1)$ is present in the previous scheme to avoid a degenerate $\langle \bar{t}t \rangle$ with $\langle \bar{b}b \rangle$. However, we can give a model in which there is: (i) a Topcolor $SU(3)$ group but (ii) no strong $U(1)$ with (iii) an anomaly-free representation content. In fact the original model of [5] was of this form, introducing a new quark of charge $-1/3$. Let us consider a generalization of this scheme which consists of the gauge structure $SU(3)_Q \times SU(3)_1 \times$

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$SU(3)_2 \times U(1)_Y \times SU(2)_L$. We require an additional triplet of fermions fields (Q_R^a) transforming as $(3,3,1)$ and $Q_L^{\dot{a}}$ transforming as $(3,1,3)$ under the $SU(3)_Q \times SU(3)_1 \times SU(3)_2$.

The fermions are then assigned the following quantum numbers in $SU(2) \times SU(3)_Q \times SU(3)_1 \times SU(3)_2 \times U(1)_Y$:

$$
\begin{aligned}
(t,b)_L \ \ (c,s)_L \ &\sim \ (2,1,3,1) \quad & Y = 1/3 \quad\quad (16) \\
(t)_R \ &\sim \ (1,1,3,1) \quad & Y = 4/3; \\
(Q)_R \ &\sim \ (1,3,3,1) \quad & Y = 0
\end{aligned}
$$

$$
\begin{aligned}
(u,d)_L \ &\sim \ (2,1,1,3) \quad & Y = 1/3 \\
(u,d)_R \ \ (c,s)_R \ &\sim \ (1,1,1,3) \quad & Y = (4/3, -2/3) \\
(\nu,\ell)_L \ \ \ell = \epsilon, \mu, \tau \ &\sim \ (2,1,1,1) \quad & Y = -1; \\
(\ell)_R \ &\sim \ (1,1,1,1) \quad & Y = -2 \\
b_R \ &\sim \ (1,1,1,3) \quad & Y = 2/3; \\
(Q)_L \ &\sim \ (1,3,1,3) \quad & Y = 0;
\end{aligned}
$$

Thus, the Q fields are electrically neutral. One can verify that this assignment is anomaly free.

The $SU(3)_Q$ confines and forms a $\langle \bar{Q}Q \rangle$ condensate which acts like the Φ field and breaks the Topcolor group down to QCD dynamically. We assume that Q is then decoupled from the low energy spectrum by its large constituent mass. There is only a lone $U(1)$ Nambu–Goldstone boson $\sim \bar{Q}\gamma^5 Q$ which acquires a large mass by $SU(3)_Q$ instantons.

1.4   Triangular Textures

The texture of the fermion mass matrices will generally be controlled by the symmetry breaking pattern of a horizontal symmetry. In the present case we are specifying a residual Topcolor symmetry, presumably subsequent to some initial breaking at some scale Λ, large compared to Topcolor, e.g., the third generation fermions in Model I have different Topcolor assignments than do the second and first generation fermions. Thus the texture will depend in some way upon the breaking of Topcolor.[7] [9]

Let us study a fundamental Higgs boson, which ultimately breaks $SU(2)_L \times U(1)_Y$, together with an effective field Φ breaking Topcolor as in eq.(4). We must now specify the full Topcolor charges of these fields. As an example,

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under $SU(3)_1 \times SU(3)_2 \times U(1)_{Y1} \times U(1)_{Y2} \times SU(2)_L$ let us choose:

$$\Phi \sim (3, \bar{3}, \frac{1}{3}, -\frac{1}{3}, 0) \qquad H \sim (1, 1, 0, -1, \frac{1}{2}) \tag{17}$$

The effective couplings to fermions that generate mass terms in the up sector are of the form

$$
\begin{aligned}
\mathcal{L}_{\mathcal{M}_U} &= m_0 \bar{t}_L t_R + c_{33} \bar{T}_L t_R H \frac{det\Phi^\dagger}{\Lambda^3} + c_{32} \bar{T}_L c_R H \frac{\Phi}{\Lambda} + c_{31} \bar{T}_L u_R H \frac{\Phi}{\Lambda} \\
&\quad + c_{23} \bar{C}_L t_R H \Phi^\dagger \frac{det\Phi^\dagger}{\Lambda^4} + c_{22} \bar{C}_L c_R H + c_{21} \bar{C}_L u_R H \\
&\quad + c_{13} \bar{F}_L t_R H \Phi^\dagger \frac{det\Phi^\dagger}{\Lambda^4} + c_{12} \bar{F}_L c_R H + c_{11} \bar{F}_L u_R H + \text{h.c.}
\end{aligned}
\tag{18}
$$

Here $T = (t, b)$, $C = (c, s)$ and $F = (u, d)$. The mass m_0 is the dynamical condensate top mass. Furthermore $det\Phi$ is defined by

$$det\Phi \equiv \frac{1}{6} \epsilon_{ijk} \epsilon_{lmn} \Phi_{il} \Phi_{jm} \Phi_{kn} \tag{19}$$

where in Φ_{rs} the first(second) index refers to $SU(3)_1$ ($SU(3)_2$). The matrix elements now require factors of Φ to connect the third with the first or second generation color indices. The down quark and lepton mass matrices are generated by couplings analogous to (18).

To see what kinds of textures can arise naturally, let us assume that the ratio Φ/Λ is small, $O(\epsilon)$. The field H acquires a VEV of v. Then the resulting mass matrix is approximately triangular:

$$
\begin{pmatrix}
c_{11}v & c_{12}v & \sim 0 \\
c_{21}v & c_{22}v & \sim 0 \\
c_{31}O(\epsilon)v & c_{32}O(\epsilon)v & \sim m_0 + O(\epsilon^3)v
\end{pmatrix}
\tag{20}
$$

where we have kept only terms of $\mathcal{O}(\epsilon)$ or larger.

This is a triangular matrix (up to the c_{12} term). When it is written in the form $U_L \mathcal{D} U_R^\dagger$ with U_L and U_R unitary and $\mathcal{D}$ positive diagonal, there automatically result restrictions on U_L and U_R. In the present case, the elements $U_L^{3,i}$ and $U_L^{i,3}$ are vanishing for $i \neq 3$, while the elements of U_R are not constrained by triangularity. Analogously, in the down quark sector $D_L^{i,3} = D_L^{3,i} = 0$ for $i \neq 3$ with D_R unrestricted. The situation is reversed when the opposite corner elements are small, which can be achieved by choosing $H \sim (1, 1, -1, 0, \frac{1}{2})$.

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These restrictions on the quark rotation matrices have important phenomenological consequences. For instance, in the process $B^0 \to \overline{B^0}$ there are potentially large contributions from top-pion and coloron exchange. However, as we show in Section IV.(B), these contributions are proportional to the product $D_L^{3;2} D_R^{3;2}$. The same occurs in $D^0 - \bar{D}^0$ mixing, where the effect goes as products involving U_L and U_R off-diagonal elements. Therefore, triangularity can naturally select these products to be small.

Selection rules will be a general consequence in models where the generations have different gauge quantum numbers above some scale. The precise selection rules depend upon the particular symmetry breaking that occurs. This example is merely illustrative of the systematic effects that can occur in such schemes.

1.5 Top-pions; Instantons; The b-quark mass.

Since the top condensation is a spectator to the TC (or Higgs) driven ESB, there must occur a multiplet of top-pions. A chiral Lagrangian can be written:

$$L = i\bar{\psi}\slashed{\partial}\,\psi - m_t(\bar{\psi}_L \Sigma P \psi_R + h.c.) - \epsilon m_t \bar{\psi} P \psi, \qquad P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (21)$$

and $\psi = (t, b)$, and $\Sigma = \exp(i\tilde{\pi}^a \tau^a / \sqrt{2} f_\pi)$. Eq.(7) is invariant under $\psi_L \to e^{i\theta^a \tau^a/2}\psi_L$, $\tilde{\pi}^a \to \tilde{\pi}^a + \theta^a f_\pi / \sqrt{2}$. Hence, the relevant currents are left-handed, $j_\mu^a = \psi_L \gamma_\mu \frac{\tau^a}{2} \psi_L$, and $< \tilde{\pi}^a | j_\mu^b | 0 >= \frac{f_\pi}{\sqrt{2}} p_\mu \delta^{ab}$. The Pagels-Stokar relation, eq.(1), then follows by demanding that the $\tilde{\pi}^a$ kinetic term is generated by integrating out the fermions. The top–pion decay constant estimated from eq.(1) using $\Lambda = M_B$ and $m_t = 175$ GeV is $f_\pi \approx 50$ GeV. The couplings of the top-pions take the form:

$$\frac{m_t}{\sqrt{2}f_\pi}\left[i\bar{t}\gamma^5 t\tilde{\pi}^0 + \frac{i}{\sqrt{2}}\bar{t}(1-\gamma^5)b\tilde{\pi}^+ + \frac{i}{\sqrt{2}}\bar{b}(1+\gamma^5)t\tilde{\pi}^-\right] \qquad (22)$$

and the coupling strength is governed by the relation $g_{bt\tilde{\pi}} \approx m_t/\sqrt{2}f_\pi$.

The small ETC mass component of the top quark implies that the masses of the top-pions will depend upon ϵ and Λ. Estimating the induced top-pion mass from the fermion loop yields:

$$m_{\tilde{\pi}}^2 = \frac{N\epsilon m_t^2 M_B^2}{8\pi^2 f_\pi^2} = \frac{\epsilon M_B^2}{\log(M_B/m_t)} \qquad (23)$$

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where the Pagels-Stokar formula is used for f_π^2 (with $k = 0$) in the last expression. For $\epsilon = (0.03, 0.1)$, $M_B \approx (1.5, 1.0)$ TeV, and $m_t = 180$ GeV this predicts $m_{\tilde{\pi}} = (180, 240)$ GeV. The bare value of ϵ generated at the ETC scale Λ_{ETC}, however, is subject to very large radiative enhancements by Topcolor and $U(1)_{Y1}$ by factors of order $(\Lambda_{ETC}/M_B)^p \sim 10^1$, where the $p \sim O(1)$. Thus, we expect that even a bare value of $\epsilon_0 \sim 0.005$ can produce sizeable $m_{\tilde{\pi}} > m_t$. Note that $\tilde{\pi}$ will generally receive gauge contributions to it's mass; these are at most electroweak in strength, and therefore of order ~ 10 GeV.

Top-pions can be as light as ~ 150 GeV, in which case they would emerge as a detectable branching fraction of top decay [11]. However, there are dangerous effects in $Z \rightarrow b\bar{b}$ with low mass top pions and decay constamnts as small as ~ 60 GeV [12]. A comfortable phenomenological range is slightly larger than our estimates, $m_{\tilde{\pi}} \gtrsim 300$ GeV and $f_\pi \gtrsim 100$ GeV.

The b quark receives mass contributions from ETC of $O(1)$ GeV, but also an induced mass from instantons in $SU(3)_1$. The instanton effective Lagrangian may be approximated by the 't Hooft flavor determinant (we place the cut-off at M_B):

$$L_{eff} = \frac{k}{M_B^2} e^{i\theta_1} \det(\bar{q}_L q_R) + h.c. = \frac{k}{M_B^2} e^{i\theta_1} [(\bar{b}_L b_R)(\bar{t}_L t_R) - (\bar{t}_L b_R)(\bar{b}_L t_R)] + h.c. \tag{24}$$

where θ_1 is the $SU(3)_1$ strong CP–violation phase. θ_1 cannot be eliminated because of the ETC contribution to the t and b masses. It can lead to induced scalar couplings of the neutral top–pion [10], and an induced CKM CP–phase, however, we will presently neglect the effects of θ_1.

We generally expect $k \sim 1$ to 10^{-1} as in QCD. Bosonizing in fermion bubble approximation $\bar{q}_L^i t_R \sim \frac{N}{8\pi^2} m_t M_B^2 \Sigma_1^i$, where $\Sigma_j^i = \exp(i\tilde{\pi}^a \tau^a / \sqrt{2} f_\pi)_j^i$ yields:

$$L_{eff} \rightarrow \frac{Nkm_t}{8\pi^2} e^{i\theta} [(\bar{b}_L b_R)\Sigma_1^1 + (\bar{t}_L b_R)\Sigma_1^2 + h.c.] \tag{25}$$

This implies an instanton induced b-quark mass:

$$m_b^\star \approx \frac{3km_t}{8\pi^2} \sim 6.6 \, k \, GeV \tag{26}$$

This is not an unreasonable estimate of the observed b quark mass as we might have feared it would be too large. Expanding Σ_j^i, there also occur induced top–pion couplings to b_R:

$$\frac{m_b^\star}{\sqrt{2} f_\pi} (i\bar{b}\gamma^5 b\tilde{\pi}^0 + \frac{i}{\sqrt{2}} \bar{t}(1 + \gamma^5) b\tilde{\pi}^+ + \frac{i}{\sqrt{2}} \bar{b}(1 - \gamma^5) t\tilde{\pi}^-) \tag{27}$$

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2 Low Energy Observables

We summarize some of the consequences of Topcolor dynamics in low energy processes. This is discussed in greater detail elsewhere.[10] Potentially large FCNC arise when the quark fields are rotated from their weak eigenbasis to their mass eigenbasis. In the case of Topcolor I, the presence of a residual $U(1)_Y$ interacting strongly with the third generation implies that the Z' will also couple to leptons in order to cancel anomalies, generating contributions to semileptonic processes. In Topcolor II the induced four–fermion interactions remain nonleptonic, hence the semileptonic processes are model dependent.

For quark field rotations are involved we must choose an anzatz for $U_{L,R}$ and $D_{L,R}$. A conservative choice is to take the squared root of the CKM matrix indicative of the order of magnitude of the effects. However, triangular textures imply the vanishing of some of the non–diagonal elements, suppressing large contributions to mixing. Presently we summarize only the largest potential effects.[10]

2.1   Semileptonic Processes

The couplings of the Topcolor I Z' to quarks are given by eq.(15). In going to the mass eigenbasis the quark fields are rotated by the matrices U_L, U_R (for the up-type left and right handed quarks) and D_L, D_R (for the down-type left and right handed quarks). We make the replacement

$$b_L \to D_L^{bb} b_L + D_L^{bs} s_L + D_L^{bd} d_L \tag{28}$$

and analogously for b_R. Thus there will be induced FCNC interactions from eq.(15).

1. $B_s \to \ell^+ \ell^-$

A flavor changing $b - s$ coupling is induced:

$$\mathcal{L}_{bsZ'} = -\frac{g_1}{2} \cot\theta' Z'^\mu \left\{ \frac{1}{3} D_L^{bb} D_L^{bs*} \bar{s}_L \gamma_\mu b_L - \frac{2}{3} D_R^{bb} D_R^{bs*} \bar{s}_R \gamma_\mu b_R + \text{h.c.} \right\}, \tag{29}$$

where the coupling to leptons is:

$$\mathcal{L}_{\ell\ell Z'} = -\frac{g_1}{2} X(\theta') Z'^\mu \left\{ (-1)\bar{\ell}_L \gamma_\mu \ell_L + (-2)\bar{\ell}_R \gamma_\mu \ell_R + \text{h.c.} \right\} \tag{30}$$

where $X(\theta') = \cot\theta'$ for $\ell = \tau$, and $X(\theta') = \tan\theta'$ otherwise.

Assuming only the Topcolor contribution, the B_s width is given by

$$\Gamma(B_s \to \ell^+\ell^-) = \frac{1}{4608\,\pi} f_{B_s}^2 m_{B_s} m_\ell^2 \sqrt{1 - \frac{4m_\ell^2}{m_{B_s}^2}} \, \delta_{bs}^2 \cot^2\theta' X^2(\theta') \left(\frac{g_1}{M_{Z'}}\right)^4 \tag{31}$$

where we've defined:

$$\delta_{bs} = D_L^{bb} D_L^{bs*} + 2 D_R^{bb} D_R^{bs*} \tag{32}$$

and B_s is the $b\bar{s}$ meson decay constant.

The most significant effect occurs in $B_s \to \tau^+\tau^-$, given that the strong $U(1)_{Y_1}$ couples to the τ lepton. This implies that $X(\theta') = \cot\theta'$. Using $f_{B_s} = 0.200$ GeV and defining:

$$\kappa_1 = \frac{g_1^2 \cot^2\theta'}{4\pi} \tag{33}$$

one gets:

$$\Gamma(B_s \to \tau^+\tau^-) = 6 \times 10^{-3} \, \delta_{bs}^2 \, \frac{k_1^2}{M_{Z'}^4} \, \text{GeV}^5 \tag{34}$$

For the neutral mixing factors we make use of the CKM squared root ansatz: ($\sqrt{\text{CKM}}$)

$$D_{L,R}^{bs} \approx 1/2 |V_{cb}| \tag{35}$$

which in this case is rather general given that (32) involves a sum of left and right contributions and, for instance, it will not vanish when the textures are triangular as in (20). We still have the freedom of the relative sign between the elements of D_L and D_R in (32). This introduces an uncertainty of a factor of 3 in the amplitude. Taking $k_1 \simeq \mathcal{O}(1)$ we get

$$BR(B_s \to \tau^+\tau^-) \approx \begin{cases} 1\ (0.1) \times 10^{-3} & \text{for } m_{Z'} = 500\text{GeV} \\ 6\ (0.7) \times 10^{-5} & \text{for } m_{Z'} = 1000\text{GeV} \end{cases} \tag{36}$$

where we have used the positive (negative) relative sign in (32). The SM prediction is $BR^{\text{SM}} \approx 4 \times 10^{-7}$, and thus these effects are potentially significant departures from the SM.

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$M_{Z'}$[GeV]	$BR(b \to s\tau^+\tau^-)$	$BR(b \to s\mu^+\mu^-)$
500	2.6×10^{-3}	8.6×10^{-6}
1000	1.6×10^{-4}	6.0×10^{-6}
SM	4×10^{-7}	5.7×10^{-6}

Table 1: Estimates of inclusive branching ratios for $b \to s\ell^+\ell^-$ in the SM and Topcolor.

2. $B \to X_s\, \ell^+\ell^-$

The dilepton mass distribution has the form

$$
\begin{aligned}
\frac{d\Gamma}{ds} &= KF(\theta')^2(1-s)^2\sqrt{1-\frac{4x}{s}}\left\{\left(|C_8|^2 + |C_8'|^2 - |C_9|^2 - |C_9'|^2\right)6x\right.\\
&\quad + \left(|C_8|^2 + |C_8'|^2 + |C_9|^2 + |C_9'|^2\right)\left[(s-4x) + \left(\left(1+\frac{2x}{s}\right)(1+s)\right]\right.\\
&\quad \left. 12C_7\mathrm{Re}[C_8 - C_8']\left(1+\frac{2x}{s}\right) + \frac{4|C_7|^2}{s}\left(1+\frac{2x}{s}\right)(2+s)\right\}
\end{aligned} \tag{37}
$$

where $s = q^2/m_b^2$ and $x = m_\tau^2/m_b^2$ and where we defined:

$$
F(\theta') = \frac{2\pi^2\, v^2}{M_{Z'}^2}\left(\frac{m_Z}{m_W}\right)^2 X^2(\theta'); \qquad K = \frac{G_F^2\alpha^2}{768\pi^5}m_b^5|V_{tb}V_{ts}^*|^2 \tag{38}
$$

and:

$$
C_8^{TC}(m_W) = -\frac{1}{2}\frac{D_L^{bs}D_L^{bb*}}{V_{tb}V_{ts}^*} \qquad C_9^{TC}(m_W) = -\frac{1}{6}\frac{D_L^{bs}D_L^{bb*}}{V_{tb}V_{ts}^*} \tag{39}
$$

$$
C_8'^{TC}(m_W) = +1\frac{D_R^{bs}D_R^{bb*}}{V_{tb}V_{ts}^*} \qquad C_9'^{TC}(m_W) = +\frac{1}{3}\frac{D_R^{bs}D_R^{bb*}}{V_{tb}V_{ts}^*} \tag{40}
$$

The Topcolor effect can be large for τ leptons due to the presence of $X(\theta') = \cot\theta'$. To illustrate the possible size of the effect we choose again the $\sqrt{\mathrm{CKM}}$ ansatz. In this case we have also to choose the sign of $D_{L,R}^{bs}$, which is taken to be positive. For $k_1 \sim \mathcal{O}(1)$ the results are given in Table I. The branching ratios are similar to those for $B_s \to \tau^+\tau^-$, given that the partial helicity suppression is balanced by the space phase suppression in the three body decay. There are no presently published limits on any of the τ channels.

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3. $B \to X_s \nu \bar{\nu}$

The decay $b \to s\ell^+\ell^-$ could have an important contribution from the τ neutrino in Topcolor models given that, in principle, they couple strongly to the $U(1)_1$. The Topcolor amplitude can be written as

$$\mathcal{A}^{TC}(b \to s\nu\bar{\nu}) = \frac{g_1^2}{4} \frac{\cot \theta' X(\theta')}{M_{Z'}^2} \{g_v \, \bar{s}\gamma_\mu b + g_a \, \bar{s}\gamma_\mu\gamma_5 b\} \, \bar{\nu}\gamma^\mu\nu_L \tag{41}$$

where

$$
\begin{aligned}
g_v &= \frac{1}{6}\left(D_L^{bb}D_L^{bs*} - 2D_R^{bb}D_R^{bs*}\right) \\
g_a &= -\frac{1}{6}\left(D_L^{bb}D_L^{bs*} + 2D_R^{bb}D_R^{bs*}\right)
\end{aligned}
$$

We take the neutral mixing to be:

$$\delta_{bs} = D_L^{bb}D_L^{bs*} = D_R^{bb}D_R^{bs*} \sim \frac{1}{2}\lambda^2 \tag{42}$$

An estimate of the ratio of branching ratios is then:[10]

$$\frac{BR^{TC}(b \to s\bar{\nu}_\tau\nu_t)}{BR^{SM}(b \to s\bar{\nu}\nu)} \sim \begin{cases} 176k_1^2 & \text{for } m_{Z'} = 500\text{GeV} \\ 11k_1^2 & \text{for } m_{Z'} = 1000\text{GeV} \end{cases} \tag{43}$$

Estimates of this mode in the SM give $BR^{SM} \simeq 7 \times 10^{-5}$.

4. $K^+ \to \pi^+ \nu\bar{\nu}$

As in $B \to X_s \nu\bar{\nu}$, we are concerned with the contact term involving τ neutrinos given that they constitute the most important Topcolor contribution. The Topcolor amplitude is given by

$$\mathcal{A}^{TC}(K^+ \to \pi^+\nu_\tau\bar{\nu}_\tau) = -\frac{g_1^2 \cot^2 \theta'}{24M_{Z'}^2} \delta_{ds} f_+(q^2)(p+k)_\mu \, \bar{\nu}_{\tau L}\gamma^\mu\nu_{\tau L} \tag{44}$$

where now $\delta_{ds}^* = D_L^{bs}D_L^{bd*} - 2D_R^{bs}D_R^{bd*}$. The form-factor $f_+(q^2)$ is experimentally well known. For one neutrino species this is given by

$$\mathcal{A}^{SM} = \frac{G_F\alpha}{\sqrt{2}\pi\sin^2\theta_W} \, f_+(q^2)(p+k)_\mu \, \bar{\nu}_L\gamma^\mu\nu_L \, \sum_j V_{js}^*V_{jd}D_j(x_j) \tag{45}$$

where $x_j = m_j^2/m_W^2$ and $D(x_j)$ is an Inami-Lim function. Since only the vector quark current contributes to the exclusive transition the Dirac structure

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in the Topcolor and SM amplitudes is the same. The ratio of the Topcolor amplitude to the SM is then

$$\frac{\mathcal{A}^{TC}}{\mathcal{A}^{SM}} = -\left(\frac{g_1 \cot\theta'}{M_{Z'}}\right)^2 \frac{\sqrt{2}\pi \sin^2\theta_W}{24\alpha \, G_F} \frac{\delta_{ds}}{\sum_j V_{js}^* V_{jd} D_j(x_j)} \tag{46}$$

For $m_t = 175$ GeV, $S \approx 2 \times 10^{-3}$. The ratio can be expressed as

$$\frac{\mathcal{A}^{TC}}{\mathcal{A}^{SM}} = -3 \times 10^9 \, \delta_{ds} \, \frac{\kappa_1}{M_{Z'}^2} \tag{47}$$

The $\sqrt{CKM}$ ansatz yields

$$\delta_{ds} = -\frac{1}{4}\lambda^5 \left(\frac{3}{4}\lambda^5\right) \tag{48}$$

when choosing positive (negative) relative signs between the two terms entering in δ_{ds}. For $M_{Z'} = 500$ GeV and $\kappa_1 = 1$ the ratio of amplitudes is about $3/2$ $(9/2)$. Therefore we must worry about the possible interference. After squaring and dividing by N_ν the total branching ratio is between $1/3$ $(16/3)$ or $4/3$ $(25/3)$ of the SM one, depending on the sign of the interference.

2.2   Nonleptonic Processes

At low energies the Topcolor interactions will induce four quark operators leading to nonleptonic processes. In Topcolor I there are potentially large corrections to B_d and B_s mixing. There are also transitions not present in the SM, most notably $b \to ss\bar{d}$, although with very small branching ratios. Perhaps the most interesting process is $D^0 - \bar{D}^0$ mixing given it is extremely suppressed in the SM. At the charm quark mass scale the dominant effect in flavor changing neutral currents is due to the flavor changing couplings of top-pions. In the case of Topcolor I the operator inducing $D^0 - \bar{D}^0$ mixing can be written as

$$\frac{m_t}{\sqrt{2}f_{\tilde{\pi}}} \, \delta_{cu} \, \bar{u}\gamma_5 c \, \bar{u}\gamma_5 c \tag{49}$$

where δ_{cu} is the factor arising from the rotation to the mass eigenstates. The contribution of (49) to the mass difference takes the form

$$\Delta m_D^{TCI} = \frac{5}{12} f_D^2 m_D \frac{m_t^2}{f_{\tilde{\pi}}^2 m_{\tilde{\pi}^0}^2} \delta_{cu} \tag{50}$$

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where f_D is the D meson decay constant. In the $\sqrt{\text{CKM}}$ ansatz and for a top-pion mass of $m_{\tilde{\pi}^0} = 200$ GeV we obtain

$$\Delta m_D^{TCI} \approx 2 \times 10^{-14} \text{ GeV} \tag{51}$$

which is approximately a factor of five below the current experimental limit. On the other hand, the SM predicts $\Delta m_D^{SM} < 10^{-15}$ GeV. This puts potentially large Topcolor effects in the discovery window of future high statistics charm experiments!

The effect could be even stronger in Topcolor II.[10] In the $\sqrt{\text{CKM}}$ ansatz this gives

$$\Delta m_D^{TCII} \approx 4 \times 10^{-12} \text{ GeV} \tag{52}$$

which violates the current experimental upper limit by about an order of magnitude. This is the single most constraining piece of phenomenology on this model. It can be avoided by taking a different ansatz for the matrices U_L and U_R. It would not be natural to have no off-diagonal elements in U_L given that it is one the factors in the CKM matrix. However U_R could be almost diagonal in which case the effect would be largely suppressed. The same is true in Topcolor I. Triangular textures in the up sector are an example of how this can be achieved. Very large effects in $B^0 - \bar{B}^0$ mixing are also avoided if triangular textures are present in the down sector.

2.3 High Energy Processes

There are other areas in which Topcolor manifests itself at higher energies, but below the Topcolor threshold. For example, there will be effects in $Z \to b\bar{b}$. Initially Zhang and Hill considered only topgluons and found positive enhancements of ΔR_b. However, a more thorough analysis of Burdman and Kominis finds that the top-pion contributions are dominant and yield a negative ΔR_b, and may impose considerable constraints on the theory [12]. Their result is interesting, but we feel not sufficiently robust for reasonable uncertainties in the strong dynamics. For example, for $f_\pi \sim 60$ GeV almost all reasonable top-pion masses are excluded, however for $f_\pi \sim 120$ GeV all top-pion masses are acceptable. We do not necessarily propose a large f_π, but this illustrates the extreme sensitivity of the result of [12] to corrections of order unity.

There are significant constraints from the ρ-parameter which have been studied by Chivukula and coauthors [13]. These evidently imply that the topgluon mass scale must significantly exceed ~ 1 TeV. A more detailed

analysis of this in the effective Lagrangian of top-pions (and other low mass boundstates) should be undertaken, in analogy to [12]. In my opinion this is potentially critical for these models.

Effects of topgluons in top production have been discussed by Hill and Parke, [14] and preliminary analysis undertaken at CDF by Harris.[15] The clear indication of the top-gluon would be a peak in the $t\bar{t}$ invariant mass at the pole, much like a Drell-Yan peak in $\mu^+\mu-$. It is possible the Tevatron lacks sufficient energy to make the full resonance, so an excess of high mass top pairs would be the next indication and the LHC would be required to produce the topgluon. The excess of Hi-p_T jets can be explained by topgluons as well, though the more conservative explanation involves a modified gluon distribution in the proton. An interesting variation on the topcolor idea, flavor democratic colorons, has been proposed to explian the Hi-p_T excess.[16]

These will be interesting issues in the Tevatron Run II.

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